

To Richard Moriarty, Director, Competition and Regulation, Postal Services Commission

From Professor Anthony Steele, Warwick Business School

Review of Deriving a Base Case NPV for Consignia, (OXERA March 20th 2002)
1 May 2002

Dear Richard,

Further to our telephone conversation, and discussion with your colleague Erik Maurice, I am setting out my initial assessment of the OXERA Report.

Their submission argues for a higher cost of capital, up from 8.7% used by Andersen to amounts in the range 15.5% - 20%. The basis of their argument is that the longer the time horizon the greater the uncertainty and hence the greater the cost of capital. The submission estimates the increase in uncertainty using a Monte-Carlo method.

The cost of capital used by Andersen is based on a standard model:

$$K = r_f + \beta \times (E(r_M) - r_f)$$

Where the parameters are :

K = cost of capital
r_f = risk free rate
β = is a measure of risk
(E(r_M) - r_f) = is the equity premium

The values used by Andersen are as follows:

r_f = 4.89%
β = 0.842
(E(r_M) - r_f) = 4.547%
giving K = 8.7%

The OXERA paper argues for a much higher parameter value for beta value.

To justify their claimed cost of capital of 15.5% would require a beta value of 2.344 and to justify a cost of capital of 20% would require a beta value of 3.338.

These beta values for equity are prima facie implausibly large.

On average the beta value for equity is unity. The London Business School Risk Measurement Service publishes monthly estimates of beta for UK listed companies. It publishes a table (attached) of the Highest and Lowest Beta. For January - March 2002 the range of values for beta were from 1.87 (Amvescap) down to .03 (Thorpe F.W.). These beta values are geared betas, that is they are measured from real companies with debt in their capital structures. Debt raises the risk of equity and thus raises these measured betas. Asset betas are estimated by de-gearing these measured betas and would have an even smaller range.

OXERA's argument in support of its own estimate of beta values is the longer the time horizon, the greater the risk, and so the greater the cost of capital. As one goes out 50 years into the future the uncertainty is high. It follows if risk is high so must be the cost of capital. As one goes out 100 years into the future the uncertainty is even higher so the risk is even higher so the cost of capital must be even higher.

Why stop at 100 years? In 1000 years the uncertainty is much higher still, so the cost of capital must be even higher! The higher the cost of capital the lower the NPV of the Free Cash Flows. If one went far enough in one's projections Consignia would be worth not billions but thousands. This reductio ad absurdum exposes why OXERA's logic is flawed.

Now this reasoning is not absurd if one were going to value a cash flow that was not due for 100 years. The problem of valuing a receipt in 100 years time is the problem that OXERA have addressed. There are two elements in valuing a long horizon cash flow: adjusting for risk and adjusting for the time value of money.

To adjust for risk the technique is to use the certainty equivalent, and then to use the risk free rate to adjust for time. What that means in practise is that one values by backward induction. One first uses a risk adjusted cost of capital to move the cash flow back one step in time from 100 years back to 99 years. This one period discounting at a risk adjusted rate delivers a certainty equivalent cash flow due in 99 years time. The risk adjusted rate has done two things, adjusted for risk and adjusted for time. This value can be moved back further in time using the risk free rate. There is no more need to further adjust for risk. The present value of a cash flow in 100 years time is thus:

$$C_{100} / (1 + \mathbf{K})(1 + \mathbf{r}_f)^{99}$$

It is clear that to use the risk adjusted rate $C_{100} / (1 + \mathbf{K})^{100}$ to value a cash flow due in 100 years is to massively undervalue.¹

The valuation model that OXERA are using is essentially a one period model. It is used mainly with monthly data in regulatory contexts. The multi-period capital asset pricing model is not extended as simply as OXERA attempt.²

¹ The reason capital budgeting practise does not apply discounting in this way, but uses the single rate to adjust for time and risk is it is usually not worth the trouble. In a long horizon valuation however it is different. See Alexander A. Robichek and Stewart C. Myers "Conceptual Problems in the Use of Risk-Adjusted Discount Rates" *Journal of Finance*. Dec 1966 pp 727-730

² Multi-period CAPM is discussed, and abandoned in Haley and Schall(chapter nine) *The Theory of Financial Decisions* (Mc Graw Hill)

Turning from the logic of their model to their attempt at estimation OXERA use Monte Carlo methods. If they had used analytical methods, such as Markov processes, the fallacy implicit in their approach would have been clearer. Monte Carlo is a black box, and OXERA's own account of how they generated randomness is incomplete. However one can examine their model by using a widely used time series process encountered in finance, that asset prices following a geometric random walk.

$$dP = P.\mu dt + P.\sigma dz$$

Where P is asset price

dP = is the change in asset price in time period dt

μ = the mean rate of growth or drift in asset prices

σ = is the standard deviation in asset price changes

dz = is a standard random term, zero mean, variance one.

Over time the long run forecast of prices after T periods, $P_T = P_0 \exp(\mu.T)$ and the risk is in this forecast is $\text{var}(P_T) = T \sigma^2$

It is clear from this analytical model that risk increases with time. The standard deviation increases with the square root of time. Using this approach merely for illustrative purposes in the manner that OXERA have done replacing σ_c by $\sqrt{T} \sigma_c$ would reveal from their section 2.3 that

$$\beta = \frac{\sigma_c \rho}{\sigma_m} \quad \text{becomes} \quad \sqrt{T} \frac{\sigma_c \rho}{\sigma_m}$$

One can see directly from an analytical model that under their assumptions the beta risk measure increases with the square root of the time horizon.

There is insufficient data disclosed of the Monte Carlo model to work out precisely how their model functions but it will have this same property, that variance increase linearly with time horizon and standard deviation with the square root of time. It follows in their approach that the cost of capital increases with the square root of time. This analytical argument supports the earlier intuitive argument that OXERA's case for an implausibly large cost of capital, is based on an implausible model.

There are two technical points on how OXERA have implemented the Monte Carlo method. They have used the method to provide range estimates, and then derived standard deviations from their range estimates by assuming that the range is equal to 3.92 standard deviations.

Monte Carlo is a poor technique for estimating ranges. For estimating means, and modes it can be pretty robust. The central limit theorem gives some comfort that the standard error of the estimate of a mean will be inversely proportional to the square root of the sample size. A Monte Carlo sample of 1000 will produce a measurement error one hundredth the size of the standard deviation in the underlying population. If one wants more accuracy then one can increase the sample size. The range

unfortunately is not so robust. It is the difference between the minimum and the maximum drawing of the random numbers. Sample size gives no increase in precision.

The technical implementation of Monte Carlo could have worked around this problem by calculation of the standard deviations directly. It could even have presented confidence intervals around these estimates. The conclusion is that even under the counterfactual that OXERA's approach to estimating the cost of capital was reasonable, then their estimates are not robust.

As I mentioned on the phone I have performed a recent cost of capital estimate for the Irish competition authorities which you may find valuable, since it reviews other regulatory determinations. The issue of cost of capital is a quagmire, but there is emerging administrative science as it is used in regulatory contexts outside the USA.

Best Regards

Anthony Steele